

The Hawking cascades of gravitons from higher-dimensional Schwarzschild black holes

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It has recently been shown that the Hawking evaporation process of $(3+1)$ -dimensional Schwarzschild black holes is characterized by the dimensionless ratio $\eta \equiv \tau_{\text{gap}}/\tau_{\text{emission}} \gg 1$, where τ_{gap} is the characteristic time gap between the emissions of successive Hawking quanta and τ_{emission} is the characteristic timescale required for an individual Hawking quantum to be emitted from the Schwarzschild black hole. This strong inequality implies that the Hawking cascade of gravitons from a $(3+1)$ -dimensional Schwarzschild black hole is extremely *sparse*. In the present paper we explore the semi-classical Hawking evaporation rates of *higher*-dimensional Schwarzschild black holes. We find that the dimensionless ratio $\eta(D) \equiv \tau_{\text{gap}}/\tau_{\text{emission}}$, which characterizes the Hawking emission of gravitons from the $(D+1)$ -dimensional Schwarzschild black holes, is a *decreasing* function of the spacetime dimension. In particular, we show that higher-dimensional Schwarzschild black holes with $D \gtrsim 10$ are characterized by the relation $\eta(D) < 1$. Our results thus imply that, contrary to the $(3+1)$ -dimensional case, the characteristic Hawking cascades of gravitons from these higher-dimensional black holes have a *continuous* character.

I. INTRODUCTION

Analyzing the dynamics of quantum fields in curved black-hole spacetimes, Hawking [1] has reached the intriguing conclusion that black holes have a well defined temperature [2, 3]. In particular, Hawking [1] has revealed that semi-classical black holes are characterized by quantum emission spectra which have distinct thermal features [4]. This remarkable prediction is certainly one of the most important outcomes of the interplay between quantum field theory and classical general relativity.

It has recently been shown [5] that the semi-classical Hawking radiation flux out of a $(3+1)$ -dimensional Schwarzschild black hole is extremely sparse [see also [6–8] for earlier discussions of this characteristic property of the $(3+1)$ -dimensional Schwarzschild black hole]. In particular, the Hawking emission of gravitons out of a $(3+1)$ -dimensional Schwarzschild black hole is characterized by the dimensionless large ratio [5]

$$\eta \equiv \frac{\tau_{\text{gap}}}{\tau_{\text{emission}}} \gg 1, \quad (1)$$

where τ_{gap} is the characteristic time gap between the emissions of successive black-hole gravitational quanta [see Eqs. (10) and (11) below], and τ_{emission} is the characteristic timescale required for an individual Hawking quantum to be emitted from the black hole [see Eq. (12) below].

The dimensionless large ratio (1) implies that the semi-classical Hawking evaporation of $(3+1)$ -dimensional Schwarzschild black holes is indeed sparse. That is, the characteristic time gap between the emissions of successive gravitational quanta out of an evaporating $(3+1)$ -dimensional Schwarzschild black hole is very large on the natural timescale $2\pi/\omega$ [see Eq. (12) below] set by the characteristic energy (frequency) of the emitted Hawking quanta. The characteristic large ratio (1) therefore suggests a simple physical picture in which an evaporating $(3+1)$ -dimensional Schwarzschild black hole [9, 10] typically emits Hawking quanta one at a time [5–8].

One naturally wonders whether the strong inequality (1), which characterizes the Hawking evaporation process of the $(3+1)$ -dimensional Schwarzschild black hole, is a generic feature of all $(D+1)$ -dimensional Schwarzschild black-hole spacetimes? In order to answer this interesting question, we shall explore in this paper the semi-classical Hawking evaporation of higher-dimensional Schwarzschild black holes. Below we shall show that the dimensionless ratio $\eta(D) \equiv \tau_{\text{gap}}/\tau_{\text{emission}}$, which characterizes the Hawking evaporation process of $(D+1)$ -dimensional Schwarzschild black holes, is a *decreasing* function of the spacetime dimension. In particular, we shall show that higher-dimensional Schwarzschild black holes with $D \gtrsim 10$ are characterized by the relation $\eta(D) < 1$. Thus, our analysis (to be presented below) reveals the fact that the Hawking cascades of gravitons from these higher-dimensional evaporating black holes have a *continuous* character.

II. THE HAWKING EVAPORATION PROCESS OF $(D + 1)$ -DIMENSIONAL SCHWARZSCHILD BLACK HOLES

We study the semi-classical Hawking emission of gravitational quanta by higher-dimensional Schwarzschild black holes. The characteristic Bekenstein-Hawking temperature of an evaporating $(D + 1)$ -dimensional Schwarzschild black hole is given by [11]

$$T_{\text{BH}}^D = \frac{(D - 2)\hbar}{4\pi r_{\text{H}}} , \quad (2)$$

where r_{H} is the black-hole horizon radius [12–15].

The radiation flux (that is, the number of quanta emitted per unit of time) and the radiation power (that is, the energy emitted per unit of time) for one bosonic degree of freedom out of a $(D + 1)$ -dimensional Schwarzschild black hole are given respectively by the integral relations [1, 16, 17]

$$\mathcal{F}_{\text{BH}}^D = \frac{1}{(2\pi)^D} \sum_j \int_0^\infty dV_D(\omega) \frac{\Gamma}{e^{\hbar\omega/T_{\text{BH}}} - 1} \quad (3)$$

and

$$\mathcal{P}_{\text{BH}}^D = \frac{1}{(2\pi)^D} \sum_j \int_0^\infty dV_D(\omega) \frac{\Gamma\hbar\omega}{e^{\hbar\omega/T_{\text{BH}}} - 1} , \quad (4)$$

where j denotes the angular harmonic indices of the emitted field mode. The frequency-dependent coefficients $\Gamma = \Gamma(\omega; j, D)$ are the dimensionless greybody factors [17] which quantify the partial scattering of the emitted field modes by the effective curvature potential that surrounds the $(D + 1)$ -dimensional Schwarzschild black hole. These factors are determined by the higher-dimensional version of the Regge-Wheeler equation [18]

$$\left(\frac{d^2}{dr_*^2} + \omega^2 - V \right) \phi = 0 , \quad (5)$$

where r_* is a ‘tortoise’ radial coordinate which is determined by the relation $dr_*/dr = [1 - (r_{\text{H}}/r)^{D-3}]^{-1}$. The effective curvature potential in the Schrödinger-like wave equation (5) is given by [18]

$$V(r; D) = \left[1 - \left(\frac{r_{\text{H}}}{r} \right)^{D-3} \right] \left[\frac{l(l + D - 2) + (D - 1)(D - 3)/4}{r^2} + \frac{(1 - p^2)(D - 1)^2 r_{\text{H}}^{D-2}}{4r^D} \right] , \quad (6)$$

where l is the angular harmonic index of the perturbation mode and $p = 0, 2$ for gravitational tensor perturbations and gravitational vector perturbations, respectively [18, 19].

Substituting into (3) and (4) the expression

$$dV_D(\omega) = [2\pi^{D/2}/\Gamma(D/2)]\omega^{D-1}d\omega \quad (7)$$

for the D -dimensional volume in frequency-space of the shell $(\omega, \omega + d\omega)$, one finds

$$\mathcal{F}_{\text{BH}}^D = \frac{1}{2^{D-1}\pi^{D/2}\Gamma(D/2)} \sum_j \int_0^\infty \Gamma \frac{\omega^{D-1}}{e^{\hbar\omega/T_{\text{BH}}} - 1} d\omega \quad (8)$$

and

$$\mathcal{P}_{\text{BH}}^D = \frac{\hbar}{2^{D-1}\pi^{D/2}\Gamma(D/2)} \sum_j \int_0^\infty \Gamma \frac{\omega^D}{e^{\hbar\omega/T_{\text{BH}}} - 1} d\omega \quad (9)$$

for the semi-classical Hawking radiation flux and the semi-classical Hawking radiation power which characterize the evaporating $(D + 1)$ -dimensional Schwarzschild black holes.

III. THE CHARACTERISTIC TIMESCALES OF THE HAWKING EVAPORATION PROCESS

An important timescale which characterizes the Hawking evaporation process of the $(D + 1)$ -dimensional Schwarzschild black holes is given by the time gap between the emissions of successive Hawking quanta. There are several distinct (though closely related) ways to quantify this fundamental time scale. Here we shall use two natural definitions for this characteristic time gap:

(1) One can use the reciprocal of the black-hole radiation flux (8) in order to quantify the characteristic time gap between the emissions of successive Hawking quanta. That is,

$$\tau_{\text{gap}}^{(1)} = \frac{1}{\mathcal{F}_{\text{BH}}^D} . \quad (10)$$

(2) One can also use the reciprocal of the black-hole radiation power (9) and the characteristic peak frequency ω_{peak} of the semi-classical Hawking radiation spectrum in order to quantify the characteristic time gap between the emissions of successive Hawking quanta. That is,

$$\tau_{\text{gap}}^{(2)} = \frac{\omega_{\text{peak}}}{\mathcal{P}_{\text{BH}}^D} . \quad (11)$$

Below we shall show that the characteristic time gaps obtained from these two definitions [Eqs. (10) and (11)] are of the same order of magnitude.

A distinct timescale which characterizes the Hawking evaporation process of the $(D + 1)$ -dimensional Schwarzschild black holes is given by the time τ_{emission} required for an individual Hawking quantum to be emitted from the evaporating black hole. This fundamental timescale can be bounded from below by the time-period it takes to the characteristic wave field emitted from the black hole to complete a full oscillation cycle [5, 10]. That is,

$$\tau_{\text{emission}} \geq \tau_{\text{oscillation}} = \frac{2\pi}{\omega_{\text{peak}}} . \quad (12)$$

Using the natural timescales (10), (11), and (12), one can define the fundamental dimensionless ratio

$$\eta^{(i)} \equiv \frac{\tau_{\text{gap}}^{(i)}}{\tau_{\text{emission}}} \quad (13)$$

which provides important information about the Hawking evaporation process of the semi-classical black holes [20]. In particular, physical situations which are characterized by the relation $\eta \gg 1$ describe Hawking evaporation processes which are extremely *sparse* (that is, the individual Hawking quanta emitted from the black hole are well separated in time), whereas physical situations which are characterized by the relation $\eta \ll 1$ describe Hawking evaporation processes which are effectively *continuous*.

In the next sections we shall investigate the functional dependence of the dimensionless ratios $\eta^{(i)}(D) \equiv \tau_{\text{gap}}^{(i)}/\tau_{\text{emission}}$ on the spacetime dimension $D + 1$ of the evaporating black-hole spacetime.

IV. THE $(3 + 1)$ -DIMENSIONAL CASE

For the emission of gravitational Hawking quanta from a $(3 + 1)$ -dimensional Schwarzschild black hole [17] one finds the characteristic dimensionless ratios

$$\eta^{(1)}(D = 3) = 5175.8 \gg 1 \quad \text{and} \quad \eta^{(2)}(D = 3) = 5371.5 \gg 1 . \quad (14)$$

As emphasized earlier, these remarkably large ratios imply that the Hawking cascade from the evaporating $(3 + 1)$ -dimensional Schwarzschild black hole is extremely *sparse* [5–8]. In other words, the dimensionless large ratios (14) imply that, on average, an evaporating $(3 + 1)$ -dimensional Schwarzschild black hole emits gravitational quanta which are well separated in time [5–8].

V. HIGHER-DIMENSIONAL BLACK HOLES: INTERMEDIATE D -VALUES

In the previous section we have seen that the $(3 + 1)$ -dimensional Schwarzschild black hole is characterized by large values of the dimensionless ratios $\eta^{(i)}(D = 3)$ [see Eq. (14)]. In the present section we shall show that the dimensionless

ratios $\eta^{(i)}(D)$, which characterize the Hawking evaporation process of $(D+1)$ -dimensional Schwarzschild black holes, are *decreasing* functions of the spacetime dimension.

The semi-classical Hawking emission of gravitons from higher-dimensional Schwarzschild black holes was investigated numerically in [18]. In Table I we display the numerically computed dimensionless ratios $\tau_{\text{gap}}^{(i)}/\tau_{\text{oscillation}}$ which characterize the Hawking emission of gravitons from higher-dimensional Schwarzschild black holes with intermediate D -values [21]. One finds from Table I that the characteristic dimensionless ratios $\tau_{\text{gap}}^{(i)}/\tau_{\text{oscillation}}$ of the $(D+1)$ -dimensional Schwarzschild black holes are *decreasing* functions of the spacetime dimension $D+1$.

In particular, we find that Schwarzschild black holes in the regime $D = O(10)$ are characterized by the dimensionless ratios $\tau_{\text{gap}}^{(i)}/\tau_{\text{oscillation}} = O(1)$. These higher-dimensional black holes therefore mark the boundary between sparse (that is, with $\eta^{(i)} \gg 1$) Hawking cascades of gravitons and continuous (that is, with $\eta^{(i)} \ll 1$) Hawking cascades of gravitons.

$D+1$	5	8	11
$\tau_{\text{gap}}^{(1)}/\tau_{\text{oscillation}}$	120.9	3.42	0.57
$\tau_{\text{gap}}^{(2)}/\tau_{\text{oscillation}}$	105.1	4.22	0.56

TABLE I: The dimensionless ratios $\tau_{\text{gap}}^{(i)}/\tau_{\text{oscillation}}$ which characterize the semi-classical Hawking emission of gravitons from $(D+1)$ -dimensional Schwarzschild black holes [18]. Here $\tau_{\text{gap}}^{(i)}$ is the characteristic time gap between the emissions of successive Hawking quanta from the higher-dimensional black hole [see Eqs. (10) and (11)] and $\tau_{\text{oscillation}}$ is the characteristic oscillation period of the emitted wave field [see Eq. (12)]. One finds that the characteristic dimensionless ratios $\tau_{\text{gap}}^{(i)}/\tau_{\text{oscillation}}$ are *decreasing* functions of the spacetime dimension $D+1$.

VI. HIGHER-DIMENSIONAL BLACK HOLES: THE LARGE- D REGIME

The D -dimensional frequency distribution $\omega^D/(e^{\hbar\omega/T_{\text{BH}}} - 1)$ [see Eq. (9)] is characterized by the peak frequency

$$\frac{\hbar\omega_{\text{peak}}}{T_{\text{BH}}^D} = D + W(-De^{-D}), \quad (15)$$

where $W(x)$ is the Lambert function. Using the small argument relation $W(x \rightarrow 0) \rightarrow 0$ and taking cognizance of the Bekenstein-Hawking temperature (2), one finds the characteristic strong inequality [22]

$$\omega_{\text{peak}} \times r_{\text{H}} = \frac{D^2}{4\pi} [1 + O(D^{-1})] \gg 1 \quad (16)$$

in the large $D \gg 1$ regime.

The strong inequality (16) reflects the fact that, for $(D+1)$ -dimensional Schwarzschild black holes in the large $D \gg 1$ regime, the characteristic wavelengths in the semi-classical Hawking emission spectrum are very *short* as compared to the curvature radius of the corresponding black-hole spacetime. As demonstrated in [22], this fact implies that the semi-classical Hawking evaporation of these higher-dimensional black holes is described extremely well by the eikonal (geometric-optics) approximation [23]. In particular, one finds [22]

$$\mathcal{F}_{\text{BH}}^D \times r_{\text{H}} = \frac{(D+1)(D-2)}{2} \left(\frac{D-2}{4\pi}\right)^D \left(\frac{D}{2}\right)^{\frac{D-1}{D-2}} \left(\frac{D}{D-2}\right)^{\frac{D-1}{2}} \frac{\zeta(D)}{\pi} \quad (17)$$

for the semi-classical Hawking radiation flux, and [22]

$$\mathcal{P}_{\text{BH}}^D \times r_{\text{H}}^2 = \frac{(D+1)(D-2)}{2} \left(\frac{D-2}{4\pi}\right)^{D+1} \left(\frac{D}{2}\right)^{\frac{D-1}{D-2}} \left(\frac{D}{D-2}\right)^{\frac{D-1}{2}} \frac{D\zeta(D+1)\hbar}{\pi} \quad (18)$$

for the semi-classical Hawking radiation power in the large- D regime [which, as explained above, corresponds to the *short* wavelengths (geometric-optics) approximation, see Eq. (16)]. Taking the asymptotic large $D \gg 1$ limit in Eqs. (17) and (18), one obtains the compact asymptotic expressions

$$\mathcal{F}_{\text{BH}}^D \times r_{\text{H}} = \frac{(4\pi)^2}{e} \left(\frac{D}{4\pi}\right)^{D+3} \quad \text{for } D \gg 1 \quad (19)$$

and

$$\mathcal{P}_{\text{BH}}^D \times r_{\text{H}}^2 = \hbar \frac{(4\pi)^3}{e} \left(\frac{D}{4\pi} \right)^{D+5} \quad \text{for } D \gg 1. \quad (20)$$

Taking cognizance of Eqs. (10), (11), (12), (19), and (20), one finds the remarkably *small* dimensionless ratios

$$\eta^{(1)}(D \gg 1) = \eta^{(2)}(D \gg 1) = \frac{e}{8\pi^2} \left(\frac{4\pi}{D} \right)^{D+1} \ll 1 \quad (21)$$

which characterize the evaporating $(D+1)$ -dimensional Schwarzschild black holes in the large- D limit [24]. These relations (and, in particular, the strong inequalities $\tau_{\text{gap}}^{(i)} \ll \tau_{\text{emission}}$) imply that the characteristic Hawking cascades from these higher-dimensional black holes have a *continuous* character.

VII. SUMMARY

It has long been known [5–8] that an evaporating $(3+1)$ -dimensional Schwarzschild black hole is characterized by an extremely *sparse* Hawking radiation flux. In particular, the dimensionless large ratios $\eta^{(i)} \equiv \tau_{\text{gap}}^{(i)}/\tau_{\text{emission}} = O(10^3)$ which characterize the $(3+1)$ -dimensional Schwarzschild black hole [see Eq. (14)] imply a simple physical picture in which the evaporating black hole typically emits Hawking quanta one at a time [5–8].

In the present paper we have analyzed the Hawking emission rates of higher-dimensional Schwarzschild black holes. It was shown that the dimensionless ratios $\eta^{(i)}(D) \equiv \tau_{\text{gap}}^{(i)}/\tau_{\text{emission}}$, which characterize the semi-classical Hawking emission of gravitons from the $(D+1)$ -dimensional Schwarzschild black holes, are *decreasing* functions of the spacetime dimension. In particular, we have shown that higher-dimensional Schwarzschild black holes with $D \gtrsim 10$ are characterized by the relation $\eta^{(i)}(D) < 1$. This fact implies that the corresponding Hawking cascades from these higher-dimensional black holes have a continuous character with the property $\tau_{\text{gap}} < \tau_{\text{emission}}$.

Moreover, we have shown that the semi-classical Hawking emission spectra of higher-dimensional Schwarzschild black holes in the large- D regime are characterized by the strong inequality [see Eq. (21)]

$$\tau_{\text{gap}}(D \gg 1) \ll \tau_{\text{emission}}(D \gg 1). \quad (22)$$

Our results therefore imply that, contrary to the $(3+1)$ -dimensional case, the characteristic Hawking cascades from these higher-dimensional black holes [25] have a *continuous* character.

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- [1] S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975).
 - [2] It is worth emphasizing that this black-hole temperature is closely related to the black-hole entropy predicted few years earlier by Bekenstein [3].
 - [3] J. D. Bekenstein, Lett. Nuov. Cim. **4**, 737 (1972); J. D. Bekenstein, Phys. Rev. D **7**, 2333 (1973); J. D. Bekenstein, Phys. Rev. D **9**, 3292 (1974).
 - [4] It is worth emphasizing that the Hawking black-hole emission spectra of evaporating semi-classical black holes are not exactly thermal. In particular, the effective curvature potential which surrounds an evaporating black hole [see Eqs. (3) and (4) below] blocks the low-energy part of the black-hole radiation spectrum.
 - [5] F. Gray, S. Schuster, A. Van-Brunt, M. Visser, e-print arXiv:1506.03975.
 - [6] J. D. Bekenstein and V. F. Mukhanov, Phys. Lett. B **360**, 7 (1995).
 - [7] J. Mäkelä, Phys. Lett. B **390**, 115 (1997).
 - [8] S. Hod, Phys. Lett. A **299**, 144 (2002) [arXiv:gr-qc/0012076].
 - [9] See [10] for the study of the fundamental dimensionless ratio $\eta \equiv \tau_{\text{gap}}/\tau_{\text{emission}}$ in the context of the semi-classical Hawking evaporation of rapidly-rotating Kerr black holes.
 - [10] S. Hod, The Euro. Phys. Jour. C **75**, 329 (2015) [arXiv:1506.05457].
 - [11] We use gravitational units in which $G = c = k_{\text{B}} = 1$.
 - [12] The horizon-radius of a $(D+1)$ -dimensional Schwarzschild black hole of mass M is given by [13] $r_{\text{H}} = [16\pi M/(D-1)A_{D-1}]^{1/(D-2)}$, where $A_{D-1} = 2\pi^{D/2}/\Gamma(D/2)$ is the generalized area of a unit $(D-1)$ -sphere.

- [13] F. R. Tangherlini, *Nuova Cimento* **27**, 365 (1963).
- [14] It is worth noting that the spherically symmetric Tangherlini black holes [13] serve here as toy models for black-hole solutions in higher dimensional spacetimes. It should be emphasized, however, that in higher-dimensional Kaluza-Klein models one finds many phases of black-hole solutions. These higher-dimensional solutions include non-uniform black strings, localized black holes, bubble-black hole sequences, and multi-black hole configurations, see [15] for an excellent review and a detailed list of references.
- [15] N. A. Obers, *Lect. Notes Phys.* **769**, 211 (2009).
- [16] W. H. Zurek, *Phys. Rev. Lett.* **49**, 1683 (1982); D. Page, *Phys. Rev. Lett.* **50**, 1013 (1983); P. Kanti, *Int. J. Mod. Phys. A* **19**, 4899 (2004).
- [17] D. N. Page, *Phys. Rev. D* **13**, 198 (1976); D. N. Page, *Phys. Rev. D* **14**, 3260 (1976).
- [18] V. Cardoso, M. Cavaglia, and L. Gualtieri, *JHEP* **0602**, 021 (2006); R. A. Konoplya and A. Zhidenko, *Phys. Rev. D* **82**, 084003 (2010); C. Harris and P. Kanti, *JHEP* **0310**, 014 (2003).
- [19] The effective curvature potential in (5) for gravitational scalar perturbations has a rather cumbersome form which is given in [18].
- [20] Using in equation (13) the average frequency ω_{ave} of the Hawking black-hole radiation spectrum instead of the characteristic peak frequency ω_{peak} of the Hawking black-hole radiation spectrum would merely change the values of the dimensionless ratios $\eta^{(i)}$ by a factor of order unity.
- [21] Note that the numerical data presented in Table I provide upper bounds on the values of the characteristic dimensionless ratios $\eta^{(i)}(D) \equiv \tau_{\text{gap}}^{(i)} / \tau_{\text{emission}}$ [see Eq. (12)].
- [22] S. Hod, *Class. Quant. Grav.* **28**, 105016 (2011) [arXiv:1107.0797]; S. Hod, *Phys. Lett. B* **746**, 22 (2015).
- [23] Note, in particular, that the effective curvature potential (6) is characterized by the asymptotic large $D \gg 1$ behavior $V_{\text{max}}(D \gg 1) = O(D^2/r_{\text{H}}^2)$. One therefore finds that the Hawking radiation spectrum is characterized by the strong inequality $\omega_{\text{peak}}^2 \gg V_{\text{max}}$ [see Eq. (16)] in the large $D \gg 1$ regime. This characteristic large-D relation implies that emitted Hawking quanta in the regime $l \ll D$ are almost unaffected by the effective scattering potential (6). This fact supports the use of the geometric-optics (short wavelengths) approximation in the asymptotic large-D regime.
- [24] It is worth noting that the large-D geometric-optics relation (21) provides the correct order of magnitude for the dimensionless ratios $\eta^{(1)}$ and $\eta^{(2)}$ already for $D = 10$. Specifically, one finds from (21) $\eta^{(1)}(D = 10) = \eta^{(2)}(D = 10) \simeq 0.42$, which is a factor ~ 1.3 smaller than the numerically computed values for the case $D = 10$, see Table I.
- [25] It is worth emphasizing again that the Hawking cascades of gravitons from $(D + 1)$ -dimensional Schwarzschild black holes in the regime $D \gtrsim 10$ are characterized by the relation $\tau_{\text{gap}}(D) < \tau_{\text{emission}}(D)$ [see Table I and Eq. (21)] and are therefore of *continuous* character.